Nonlocal chaotic phase synchronization

Meng Zhan,[1] Zhi-gang Zheng,[1] Gang Hu,[2,1] and Xi-hong Peng[1]
[1]*Department of Physics, Beijing Normal University, Beijing 100875, China*
[2]*China Center for Advanced Science and Technology (CCAST) (World Laboratory), P.O. Box 8730, Beijing 100080, China*
(Received 21 March 2000; revised manuscript received 19 May 2000)

A novel synchronization behavior, nonlocal chaotic phase synchronization, is investigated. For two coupled Rossler oscillators with only one forced by an injected periodic signal, the phase of the unforced oscillator can be locked to the phase of the periodic signal while the forced one is well unlocked by the signal; in a chain of coupled chaotic oscillators with nearest coupling, the phase of an oscillator (or a cluster) can be locked to another nonneighbor one. Moreover, the mechanism underlying the transition to nonlocal synchronization is discussed in detail.

PACS number(s): 05.45.−a

Synchronization is a basic phenomenon in physics, discovered by Huygens at the beginning of the modern age of science [1]. In the classical sense, synchronization means frequency and phase locking of periodic oscillators. Recently, the notion of ''phase synchronization'' has been extended to chaotic systems, and scientists have extensively studied not only the phase synchronizations between chaotic oscillators with external periodic drivings [2–6], but also that of the coupled oscillator systems [7–12]. Phase synchronization is an intrinsic feature in the relation between the coupled oscillators (or between oscillators with injected signals), and this feature gives essential influence to the system dynamics.

For a chaotic system of coupled oscillators with nearest coupling whose natural frequencies are not equal, the intuitive idea for phase synchronization is the following: due to the nearest-coupling nature, some neighbor oscillators should first form synchronous clusters, then by increasing coupling these clusters develop from near to far through neighbor aggregation and produce larger clusters. Finally, full synchronization can be established through neighbor cluster merging. This physical picture has been clearly shown in Refs. [9] and [10] by using diagrams of synchronization plateaus and bifurcation trees.

Nevertheless, in Ref. [10], some of us found a novel kind of phase synchronization, i.e., an oscillator can be synchronized to a next-to-the-nearest-neighbor oscillator by a nonlocal synchronization, while the oscillator in between is not synchronized to its two neighbors. This observation is strikingly contrary to our intuition. However, the finding there was occasional. We did not know whether this nonlocal synchronization is popular, whether we can find nonlocal synchronization between clusters, and whether the nonlocal synchronization within larger spatial distance is possible. In particular, we did not understand the mechanism underlying this kind of nonlocal synchronization phenomena. This paper is aiming to answer the above problems, by considering the coupled Rossler systems as our model.

First, we investigate a simple system with two coupled nonidentical Rossler oscillators (No. 1 and No. 2 whose natural frequencies are not equal) with No. 2 forced by a periodic signal. It should be emphasized that the No. 1 oscillator is coupled to the No. 2 one, but not connected with the periodic forcing directly. Then any synchronization between the No. 1 oscillator with the signal can be made only through the dynamic variable of the No. 2 oscillator. Our interest rests in whether phase synchronization between the signal and the unforced oscillator (No. 1) can be established while the forced one (No. 2) is in a desynchronized situation. The model reads

$$\begin{aligned}\dot{x}_1&=-w_1y_1-z_1+e(x_2-x_1),\\ \dot{y}_1&=w_1x_1+0.15y_1,\\ \dot{z}_1&=0.2+z_1(x_1-10.0),\\ \dot{x}_2&=-w_2y_2-z_2+e(x_1-x_2),\\ \dot{y}_2&=w_2x_2+0.15y_2+A\sin(\Lambda t),\\ \dot{z}_2&=0.2+z_2(x_2-10.0),\end{aligned} \tag{1}$$

where subscripts 1 and 2 represent the unforced and forced oscillators, respectively, w_1 and w_2 are the natural frequencies of the two oscillators, e is the coupling coefficient between them, Λ is the forcing frequency, and A is the driving intensity.

For a Rossler oscillator, we can define its average frequency (the rotation number) as [4]

$$\Omega_i=\langle d\theta_i(t)/dt\rangle=\lim_{T\to\infty}\frac{1}{T}\int_0^T\dot{\theta}_i(t)dt, \tag{2}$$

based on the phase definition of

$$r_i(t)=\sqrt{x_i(t)^2+y_i(t)^2},\quad \theta_i(t)=\arctan\left(\frac{y_i(t)}{x_i(t)}\right),\quad i=1,2. \tag{3}$$

To show nonlocal synchronization clearly, we fix the natural frequencies of the two Rossler oscillators w_1 and w_2 to $w_1=1.0$, $w_2=0.65$, which stay far away each other. In Fig. 1 we take $A=1.0$, $e=0.1$, change the driving frequency Λ from 0.97 to 1.03, and plot the rotation numbers Ω_1/Λ and Ω_2/Λ vs Λ, respectively. From the flat plateau in Fig. 1(a) we can clearly see that the No. 1 oscillator Ω_1 is locked

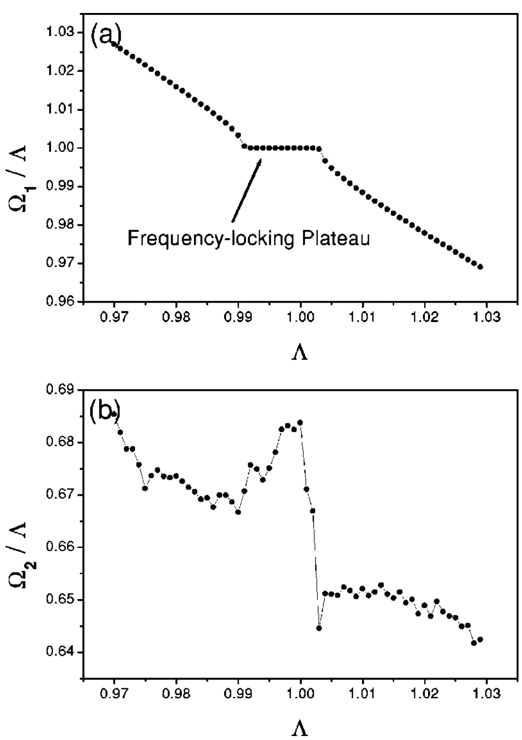

FIG. 1. (a) and (b) Ω_1/Λ and Ω_2/Λ plotted vs Λ, respectively. $w_1=1.0$, $w_2=0.65$, $A=1.0$, $e=0.1$. There is a plateau at $\Omega_1/\Lambda=1$ for $0.991<\Lambda<1.003$ in (a), where nonlocal frequency locking occurs between the signal and the No. 1 site that is not directly forced.

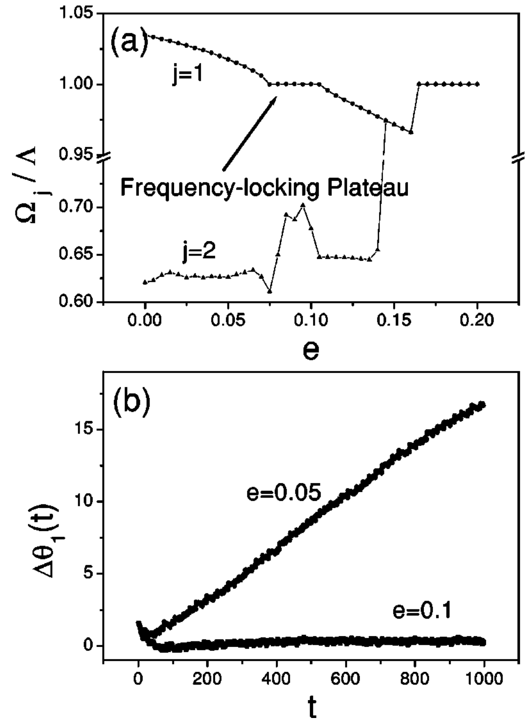

FIG. 2. $w_1=1.0$, $w_2=0.65$, $A=1.0$, $\Lambda=1.0$. (a) Ω_j/Λ vs e (j is 1 and 2). Nonlocal frequency locking occurs for $0.075<e<0.105$, and the coupled system stays at chaotic state. (b) $\Delta\theta_1(t)=\theta_1(t)-\Lambda t$ plotted vs t for $e=0.05$ (no synchronization) and $e=0.1$ (synchronization). Phase locking between the signal and the No. 1 site is clearly observed on the frequency-locking plateau.

to the forcing frequency Λ, though it is not driven directly by the injected signal. On the contrary, in Fig. 1(b) the average frequency of No. 2, Ω_2, is well desynchronized from Λ. In Fig. 2(a) we plot Ω_j/Λ vs e (j is 1 and 2), and find a frequency-locking plateau $\Omega_1/\Lambda=1$ for $0.075<e<0.105$ while the coupled system well stays at chaotic state. In Fig. 2(b) we plot the phase difference between the No. 1 oscillator and the driving force $\Delta\theta_1(t)=\theta_1(t)-\Lambda t$, for the parameters before ($e=0.05$) and on the frequency-locking plateau ($e=0.1$). Before frequency locking, $|\Delta\theta_1(t)|$ increases linearly with certain oscillation, while on the frequency-locking condition we find phase locking, i.e., $\Delta\theta_1(t)$ fluctuates around a certain finite value. We call this situation nonlocal phase synchronization.

It is interesting to investigate why the unforced site (site 1) can be locked to the signal under the condition that the forced site (site 2) and also the coupling input from site 2 to site 1 [i.e., ex_2 in the first equation of Eqs. (1)] are not synchronized to the injecting signal. In Fig. 3(a) we plot the spectrum of the x_1 variable in the nonlocal synchronization situation, which shows a single huge peak at f ($f=\Lambda/2\pi$). It should be noted that the frequency f_1, which is the frequency of site 1 when driving is absent, is a bit away from f, and then no synchronization can be expected without the coupling between sites 1 and 2. In Fig. 3(b) we plot the spectrum of x_2 for the same parameters of Fig. 2(a), and find that site 2 has a huge spectrum peak far from the signal frequency f, indicating desynchronization. Nevertheless, there is a small spectrum peak at f induced by the injecting signal. This small peak shows that a small component of the injected spectrum is produced in the output of the desynchronized forced site. It is just this small component that plays a key role for the nonlocal phase synchronization, i.e., through the coupling, this component drives the site 1 to shift its frequency and induces the phase synchronization between site 1 and the signal. In Figs. 3(c) and 3(d) we do the same as 3(a) and 3(b), respectively, by moving Λ away from the synchronization region, and it is clear that both x_1 and x_2 are desynchronized from the injecting signal.

Above we have investigated a model of two coupled Rossler oscillators with one driven by an external periodic signal and have found the nonlocal synchronization between the unforced oscillator and the injected signal. Now we come to autonomous systems of coupled Rossler oscillators and study the possible mutual nonlocal synchronization. The model reads

$$\dot{x}_j=-w_jy_j-z_j+e(x_{j+1}+x_{j-1}-2x_j),$$

$$\dot{y}_j=w_jx_j+0.15y_j,$$

$$\dot{z}_j=0.2+z_j(x_j-10.0),\quad (j=1,\ldots,N), \tag{4}$$

where nearest coupling is considered, e represents the diffusive coupling coefficient, N is the number of oscillators, and w_j are the natural frequencies of the coupled oscillators, which are random numbers in some scope. We use a periodic boundary condition.

We start with considering $N=5$. In Fig. 4, we get a typical bifurcation tree revealing the various synchronizations between the oscillators by varying e from a small value to a

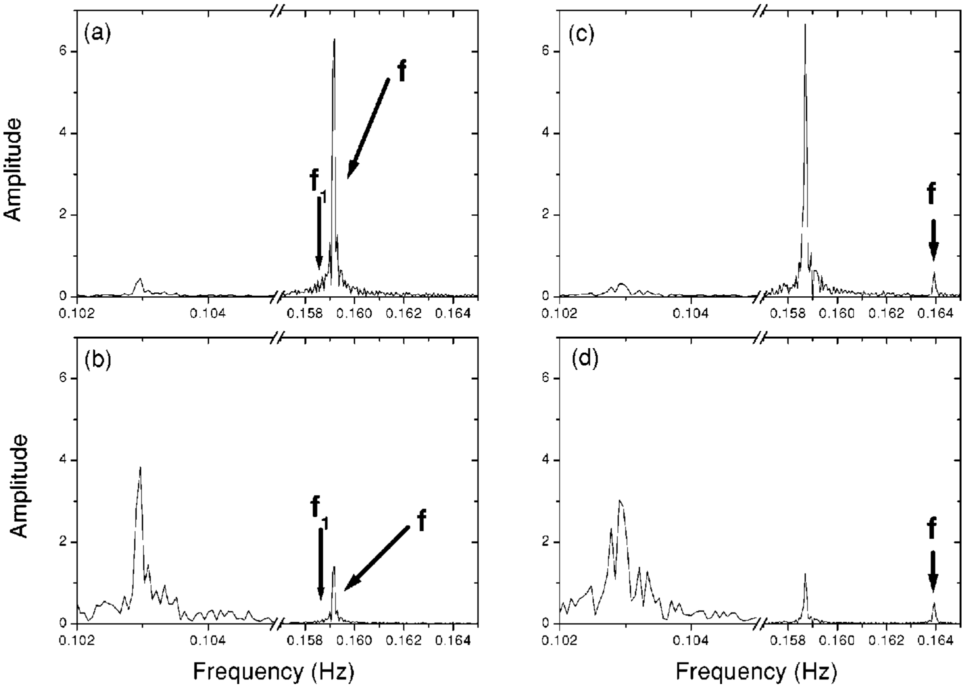

FIG. 3. (a) and (b) The spectra of x variables of sites 1 and 2, respectively. $\Lambda=1.0$, which is inside of the nonlocal synchronization region of Fig. 1(a). The arrows f_1 indicate the spectrum peak of site 1 without driving. The arrows f ($f=\Lambda/2\pi$) represent the frequency of forcing. Both are not equal to each other. (c) and (d) The same as (a) and (b) except $\Lambda=1.03$, which is outside of the nonlocal synchronization region.

large one. In particular, at the parameter interval $0.04<e<0.05$, a two-site cluster (2, 3) synchronizes with the site 5 nonlocally. In Figs. 5(a) and 5(b) we plot $\Delta\theta_{j,5}(t)=\theta_j(t)-\theta_5(t)$ off and at nonlocal synchronization, respectively. A phase locking between the two nonlocal oscillators is clearly shown in Fig. 5(b), moreover, we find that while sites 3 and 5 are synchronized to each other, the site in between, i.e., No. 4 is not in synchronization with them.

In Figs. 6(a), 6(b), 6(c), and 6(d) we show logarithms of the spectra of all five oscillators at $e=0.01$, $e=0.03$, $e=0.045$, and $e=0.1$, respectively. The rotation numbers of the oscillators are determined by their highest spectrum peak, then certain phase synchronizations appear if some of the highest peaks of coupled oscillators stay at a same location. When the coupling intensity is small, all the oscillators take frequencies near their natural frequencies and remain desynchronized [Fig. 6(a)]. As e increases, some nearest oscillators

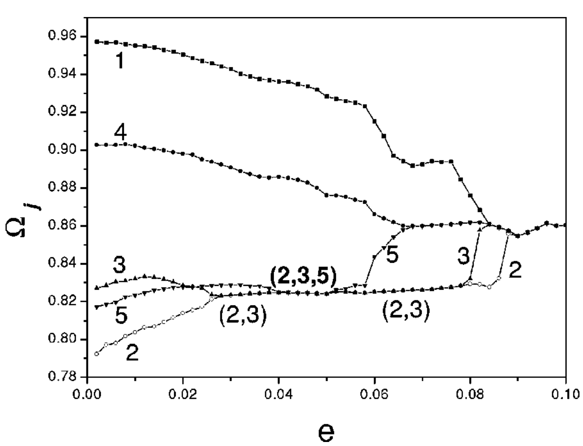

FIG. 4. $N=5$, the bifurcation tree of coupled chaotic Rossler oscillators, whose natural frequencies are random numbers, indicated in the figure at $e=0$. In the region $0.04<e<0.05$, nonlocal synchronization between a cluster (2,3) and an oscillator (5) emerges.

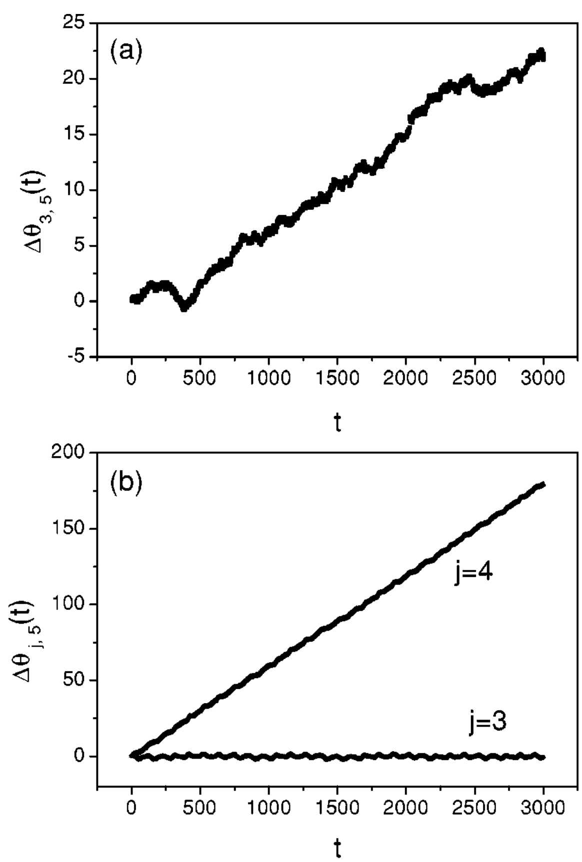

FIG. 5. (a) $\Delta\theta_{3,5}(t)=\theta_3(t)-\theta_5(t)$ plotted vs t, $e=0.01$, it is clear that sites 3 and 5 are desynchronized. (b) $\Delta\theta_{j,5}(t)=\theta_j(t)-\theta_5(t)$ ($j=3$ and 4) plotted vs t, $e=0.045$, so nonlocal synchronization between sites 3 and 5 is obvious while the middle site 4 is not in synchronization state under the same condition.

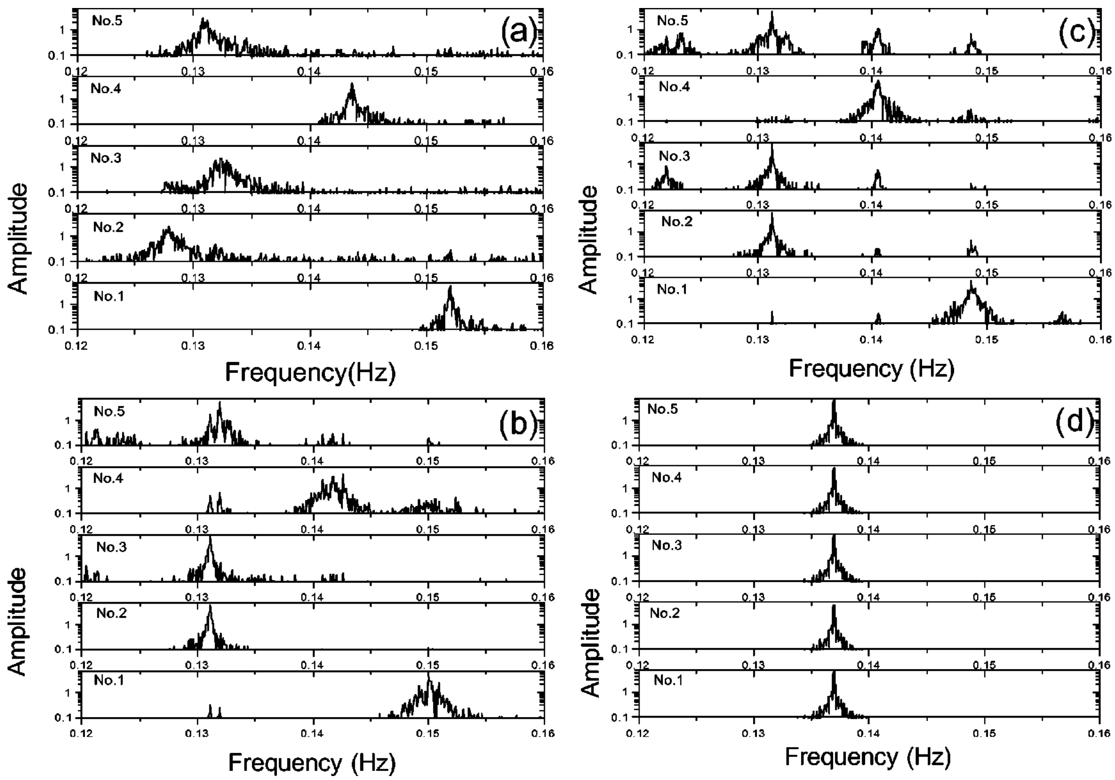

FIG. 6. The spectra of all the five oscillators of Fig. 4 at different coupling coefficients (a) $e=0.01$, (b) $e=0.03$, (c) $e=0.045$, and (d) $e=0.1$.

having near frequencies (e.g., sites 2, 3) get to be synchronized to make a cluster [Fig. 6(b)], so the corresponding peaks move to a same location in the spectrum figure. For certain coupling intensity, the nonlocal oscillators with close (but not equal) nature frequencies can move their main spectrum peaks to the same position, leading to the same rotation number, i.e., nonlocal synchronization. This situation can be clearly seen in Fig. 6(c) (sites 2, 3, and 5). An interesting point is that as the nonlocal phase synchronization occurs, the spectra of the oscillators between the synchronized ones show small peaks at the synchronous frequency though their main peaks are away from it (see the spectrum of No. 1). These small synchronous components play the key role of bridges leading to the synchronization between the nonlocal oscillators. By further increasing the coupling e, the system undergoes complicated synchronization and desynchronization transitions as shown in Fig. 4, and finally reaches full synchronization for sufficient large e. This is the case of Fig. 6(d). Moreover, it is observed that the system motion in the whole coupling range of nonlocal synchronization in Fig. 4, $0.04<e<0.05$, is chaotic, then we are considering chaotic synchronization.

In Figs. 4–6, we find nonlocal synchronization in a weak sense, i.e., the synchronization occurs between a small cluster (2, 3) and a single oscillator (5); the nonlocal distance is only a single site. It is interesting to detect the possibility for more general nonlocal synchronization, e.g., the nonlocal synchronization between large clusters and over large distance. In Fig. 7(a) we take $N=15$, and again randomly choose the natural frequencies of the coupled oscillators. We change e from 0.0 to 0.45 and plot the rotation numbers Ω_j in Fig. 7(a). For this many-body system, we find indeed non-

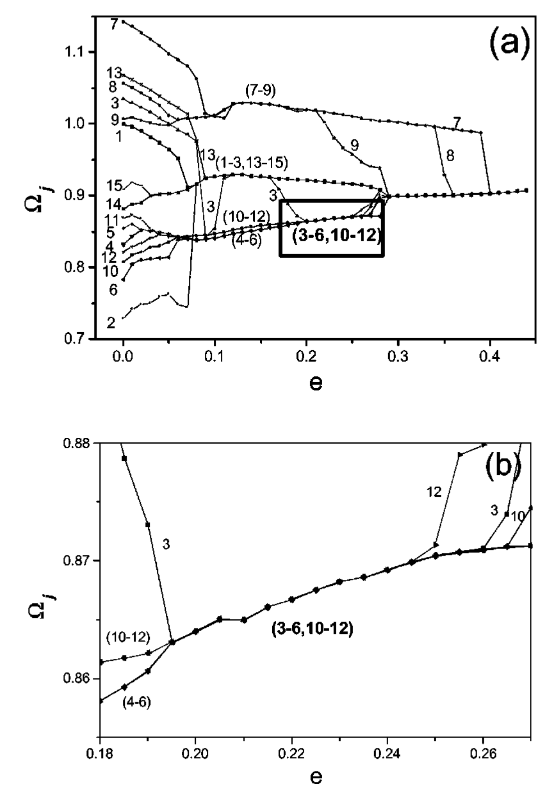

FIG. 7. (a) The same as Fig. 4 except $N=15$. In the region $0.195<e<0.245$, nonlocal synchronization between two clusters (3–6) and (10–12) appears. Moreover, the distance between these two synchronized clusters is 3-site. (b) The blowup of the rectangle region of (a).

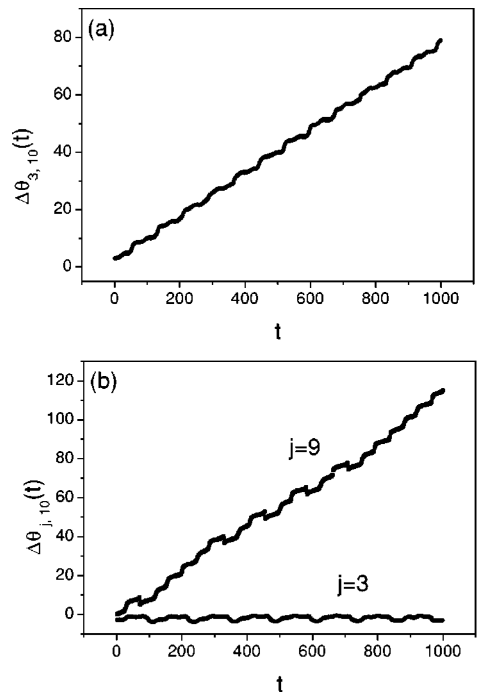

FIG. 8. The same as Fig. 5 with 15-site system [Fig. 7] is considered. (a) $e=0.125$. (b) $e=0.225$. Nonlocal synchronization between sites 3 and 10 is observed, which belong to two separated synchronized clusters, while the sites in between (e.g., site 9) are not in synchronization status.

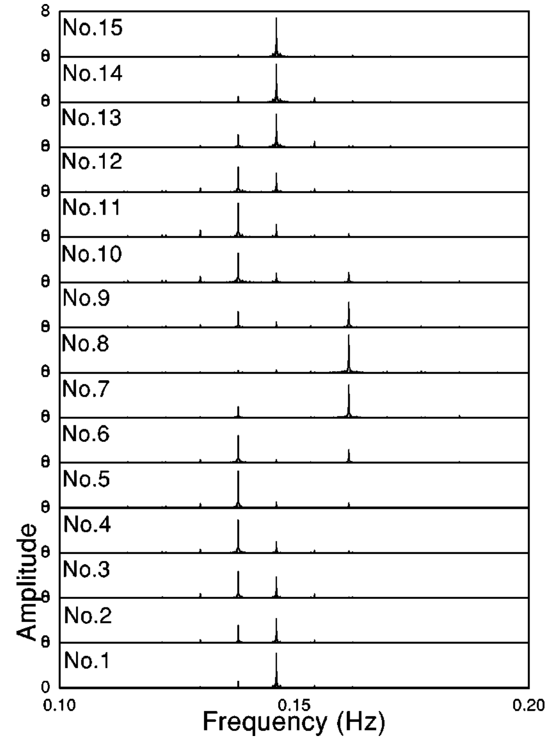

FIG. 9. The spectra of x variables of all 15 oscillators. $e=0.225$, which is in the nonlocal synchronization region of Fig. 7. Note the small synchronization spectrum components in the spectra of nonsynchronized sites 1, 2, 7–9, 13–15, which play the role of transferring nonlocal synchronization.

local phase synchronization between two large clusters, i.e., the clusters (3–6) and (10–12), and the distance between these two clusters reaches three sites. In order to make the above conclusion more convincing, in Fig. 7(b) we amplify the rectangle region of Fig. 7(a), then the nonlocal synchronization of two large clusters is shown without any ambiguity. In Fig. 8 we do the same as Fig. 5 with the nonlocal synchronization of the 15-site system (Fig. 7) being considered. Again, clear phase locking of nonlocal sites (sites 3 and 10) is justified while some sites (e.g., site 9) in between are well desynchronized.

In Fig. 9 we plot the spectra of x variables of all of the 15 oscillators of the system by fixing $e=0.225$, which is right in the nonlocal synchronization region of Fig. 7. The spectra of 3–6 and 10–12 sites have the main spectrum peaks exactly at the same synchronous frequency. On the other hand, the spectra of all other nonsynchronized sites between the two synchronized clusters have small components at the synchronous frequency, and these components fulfill the task of transferring the synchronization between the two distantly separated clusters.

From Figs. 4 and 7, it is clear that the nonlocal synchronizations take place in a certain intermediate range of coupling intensity, and both too small and too large couplings will destroy this kind of synchronization. Too small coupling produces too weak signals, which are not sufficient for transferring synchronization between the corresponding nonlocal sites, while too large coupling can bring the intermediate sites into synchronizations, and then change the nonlocal synchronizations to local ones. We have investigated more general cases, such as the coupled Rossler system with larger N and with both diffusive and gradient couplings, and found nonlocal synchronizations can be often observed, depending on the distributions of natural frequencies of the oscillators.

In conclusion, we have investigated the nonlocal phase synchronization of chaotic oscillators in detail. This synchronization can occur between an injected signal and oscillators not directly forced, and can also occur between nonneighbor oscillators. Even the nonlocal synchronization can occur between chaotic clusters, with a relatively large spatial distance. The mechanism underlying this seemly strange synchronization has been very clear: the synchronous components can be transferred among the nonsynchronized sites, which produce the nonlocal synchronization of sites or clusters that are a distance apart.

This research was supported by the National Natural Science Foundation of China, the Nonlinear Science Project of China, and the Foundation of Doctoral Training of the Educational Bureau of China. Z. Z. was supported by the Special Funds for Major State Basic Research Projects and by the Foundation for Excellent Teachers from the Educational Bureau of China.

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